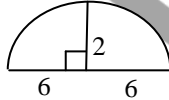


**DEFENCE SERVICES ACADEMY
2016-2017 ENTRANCE EXAMINATION
MATHEMATICS TEST**

Time Allowed: 2 Hours

PART (A)

1. Choose the correct or the most appropriate answer for each question.
Write the letter of the correct or the most appropriate answer. **(22 Marks)**
- (1) An operation \otimes is defined by $x \otimes y = \frac{3xy}{x+y}$, then the value of x for which $x \otimes 2x = 4$ is
A. -3 B. 3 C. 1 D. -1 E. 2
- (2) ${}^n C_0 + {}^n C_1 + {}^n C_n =$
A. n B. $n+1$ C. 2 D. $n+2$ E. none of these
- (3) The solution set in \mathbb{R} for the inequation $-3x^2 \leq 0$ is
A. $\{x | x > 0\}$ B. \mathbb{R} C. \emptyset D. $\{x | x \leq 0\}$ E. $\{x | x < 0\}$
- (4) Given that $3, x, y, z, 23, \dots$ is an arithmetic sequence, then $z =$
A. 13 B. 18 C. 21 D. 22 E. none of these
- (5) Determinant of the matrix $\begin{pmatrix} \sin 2\theta & \sin 2\theta \\ -\cos 2\theta & \cos 2\theta \end{pmatrix}$ is
A. 1 B. $\sin 4\theta$ C. $\cos 4\theta$ D. $\sin 2\theta$ E. $\cos 2\theta$
- (6) If A is an event such that $P(A) = y$ and $P(\text{not } A) = x$, then $x^3 + y^3 =$
A. $3xy$ B. $1+3xy$ C. $3xy-1$ D. $1-3xy$ E. none of these
- (7) The arc forms part of a circle whose radius is
A. 9 B. 10 C. 16
D. 18 E. 20
- 
- (8) In $\triangle ABC$, P and Q are two points on the sides AB and AC respectively. If $PQ \parallel BC$ and $\alpha(\triangle APQ) : \alpha(\text{BCQP}) = 9 : 16$, then $AP : PB$ is
A. 3 : 4 B. 4 : 3 C. 3 : 5 D. 5 : 3 E. 3 : 2
- (9) $ABCD$ is a parallelogram. If L and M are the middle points of BC and CD , then $\overrightarrow{AL} + \overrightarrow{AM} =$
A. $\frac{1}{2}\overrightarrow{AC}$ B. $\frac{3}{2}\overrightarrow{AC}$ C. \overrightarrow{AC} D. $\frac{1}{4}\overrightarrow{AC}$ E. none of these
- (10) $\tan 675^\circ =$
A. 0 B. $\frac{1}{\sqrt{3}}$ C. 1 D. -1 E. none of these
- (11) The gradient of normal line to the curve $y = 2\sqrt{x}$ at the point $x = 9$ is
A. $\frac{1}{3}$ B. $-\frac{1}{3}$ C. 3 D. -3 E. 6

P.T.O. \longrightarrow

PART (B)

2. (a) Functions f and g are defined by $f(x) = \frac{x}{2-x}$, $x \neq 2$ and $g(x) = ax + b$. Given that $g^{-1}(7) = 3$ and $(g \circ f)(5) = -7$, calculate the values of a and b .
(6 marks)
- (b) The polynomial $px^3 + qx^2 - 5x - 6$ is exactly divisible by $2x^2 + x - 6$. Calculate the values of p and q , and factorize the polynomial completely.
(7 marks)
3. (a) Given that $\left(a - \frac{x}{2}\right)^6 = b - 96x + cx^2 + \dots$, find a , b , c .
(6 marks)
- (b) Find the solution set in \mathbb{R} of the inequation $(x-6)^2 > x$ by graphical method and illustrate it on the number line.
(7 marks)
4. (a) An A.P. contains thirteen terms. If the sum to first four terms is 32 and the sum of the last four terms is 176, find the middle term of that A.P..
(6 marks)
- (b) Find the solution set of the system of equations $3x - 7y = 35$, $x + y = 5$, by matrix method; the variables are on the set of real numbers.
(7 marks)
5. (a) How many 3-digit numerals can you form from 2, 5, 6 and 0 without repeating any digit? Find the probability of an odd number and find the probability of a numeral which is a multiple of 5.
(6 marks)
- (b) If L , M , N be the middle points of the sides of a $\triangle ABC$, and if P , Q , R be the feet of the perpendiculars from the vertices on the opposite sides, prove that P, N, Q, L, M, R are concyclic.
(7 marks)
6. (a) In $\triangle ABC$, AD and BE are the altitudes. If $\alpha(\triangle DEC) = \frac{3}{4}\alpha(\triangle ABC)$, prove that $\angle ACB = 30^\circ$.
(6 marks)
- (b) The position vectors of points P, Q and R relative to an origin O are $3\vec{b} + 5\vec{c} - 2\vec{a}$, $7\vec{a} - \vec{c}$ and $\vec{a} + 2\vec{b} + 3\vec{c}$ respectively. Show that P, Q and R are collinear. Show also that $\vec{PQ} = \vec{QR} + \vec{PR}$.
(7 marks)
7. (a) Given that $\frac{\cos(\alpha - \beta)}{\cos(\alpha + \beta)} = \frac{7}{5}$, prove that $\cos \alpha \cos \beta = 6 \sin \alpha \sin \beta$ and deduce a relationship between $\tan \alpha$ and $\tan \beta$. Given further that $\alpha + \beta = 45^\circ$, calculate the value of $\tan \alpha + \tan \beta$.
(6 marks)
- (b) Find the minimum value of the sum of a positive number and its reciprocal.
(7 marks)