

## DEFENCE SERVICES ACADEMY

## ENTRANCE EXAMINATION FOR THE D.S.A. CADETS

MATHEMATICS

ANSWER ALL QUESTIONS

Time Allowed (2) hours

## PART (A)

1. Choose the correct or the most appropriate answer for each question.

Write the letter of the correct or the most appropriate answer.

- (1)  $\odot$  is defined on the set of real numbers by  $(a-b)\odot(a+b) = ka^2 + b$ . If  $6\odot 8 = 50$  then  $k =$   
 A. 2                      B. -2                      C. -1                      D. 1                      E. 0
- (2) If  $x^2 - 5x + 6$  is a factor of a polynomial  $f(x)$ , which of the following is certainly true?  
 A.  $f(6) = 0$       B.  $f(-3) = 0$       C.  $f(1) = 0$       D.  $f(-2) = 0$       E.  $f(3) = 0$
- (3)  $n$  is a positive integer and in the expansion of  $(1+x)^n$  in ascending powers of  $x$ , the coefficient in the 4<sup>th</sup> term is  $k$ . Then the coefficient in the 5<sup>th</sup> term is  
 A.  $\frac{(n-2)k}{4}$       B.  $\frac{(n-2)k}{5}$       C.  $\frac{(n-3)k}{4}$       D.  $\frac{(n-3)k}{5}$       E.  $\frac{(n-4)k}{5}$
- (4) If 5,  $x$ ,  $y$ , 38 are the first four terms of an A.P., then  $y - x$  is  
 A. 9                      B. 10                      C. 11                      D. 12                      E. 13
- (5) If  $\begin{pmatrix} a \\ b \end{pmatrix} + 2 \begin{pmatrix} 6 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ , then  $a - b =$   
 A. -2                      B. -1                      C. 0                      D. 1                      E. 2
- (6) If  $X$  and  $Y$  are two events such that  $P(X) = \frac{3}{5}$  and  $P(\text{not } Y) = \frac{2}{7}$ , then  $P(\text{not } X) + P(Y) =$   
 A.  $\frac{2}{5}$                       B.  $\frac{5}{7}$                       C.  $\frac{31}{35}$                       D.  $\frac{39}{35}$                       E. None of these
- (7) In a cyclic quadrilateral  $ABCD$ ,  $\angle A = 25^\circ$ ,  $\angle B = 60^\circ$ . Then  $\angle C - \angle D =$   
 A.  $35^\circ$                       B.  $40^\circ$                       C.  $45^\circ$                       D.  $50^\circ$                       E.  $55^\circ$
- (8) The sum and difference of areas of two similar triangles are 234 and 90. The ratio of corresponding sides is  
 A. 5 : 8                      B. 9 : 16                      C. 4 : 5                      D. 2 : 3                      E. 3 : 5
- (9)  $ABCD$  is a parallelogram. Then  $\overline{AC} + \overline{BD} =$   
 A.  $\overline{AB}$                       B.  $2\overline{AB}$                       C.  $\overline{BC}$                       D.  $2\overline{BC}$                       E.  $\overline{BA}$
- (10)  $\cos^2 330^\circ =$   
 A.  $\frac{4}{3}$                       B.  $\frac{3}{4}$                       C.  $\frac{3}{2}$                       D.  $\frac{2}{3}$                       E. 1
- (11) The stationary point of the curve  $y = x(3-x)$  is  
 A.  $\left(\frac{3}{2}, \frac{9}{4}\right)$       B.  $\left(\frac{9}{4}, \frac{3}{2}\right)$       C.  $(0, 0)$       D.  $\left(\frac{2}{3}, \frac{4}{9}\right)$       E. None of these

(22) Marks

P. T. O.  $\rightarrow$

## PART (B)

2. (a) Functions  $h$  and  $g$  are defined by  $g : x \mapsto \frac{x+1}{x-2}$ ,  $x \neq 2$ ,  $h : x \mapsto \frac{ax+3}{x}$ ,  $x \neq 0$ . Find

The value of  $a$  for which  $(h \circ g^{-1})(4) = g^{-1}(2)$ . (6 marks)

- (b) Find the values of  $p$  and  $q$  for which the expression  $12x^4 + 16x^3 + px^2 + qx - 1$  is divisible by  $4x^2 - 1$ . Hence, find the other factors of the expression. (7 marks)

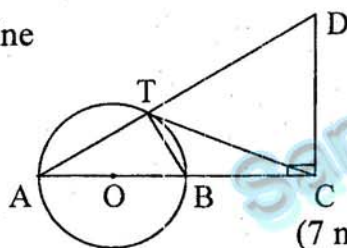
3. (a) In the expansion of  $(3+4x)^n$ , the coefficients of  $x^4$  and  $x^5$  are in the ratio 3:4. Find the value of  $n$ . Calculate the ratio of the coefficients of  $x^5$  and  $x^6$ . (6 marks)

- (b) The sum of the first three terms of a G.P. is 63 and the sum of the 4<sup>th</sup>, 5<sup>th</sup> and 6<sup>th</sup> terms is  $-\frac{7}{3}$ . Find the sum to infinity of the G.P. (7 marks)

4. (a) By using the definition of the inverse of a matrix, find the inverse of  $M = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix}$ . (7 marks)

- (b) If two dice are tossed, find the probability of getting a total of 10 or more, and calculate the probability of both dice show the same number. (6 marks)

5. (a) In circle  $O$ , the diameter  $AB$  is produced to  $C$  and the line  $CT$  is the tangent to the circle at  $T$ . The line drawn perpendicular to  $AC$  at  $C$  meets  $AT$  produced at  $D$ .

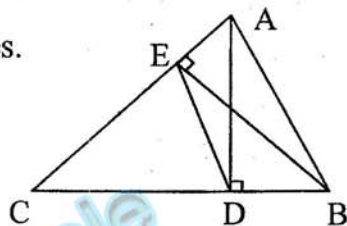


Prove that  $BCDT$  is cyclic quadrilateral and  $CT = CD$ . (7 marks)

- (b) In  $\triangle ABC$ ,  $AD$  and  $BE$  are the altitudes.

$$\text{If } 4\alpha(\triangle DEC) = 3\alpha(\triangle ABC),$$

find  $\angle ACB$ .



(6 marks)

6. (a) Points  $A$  and  $B$  have position vectors  $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$  respectively, relative to an origin  $O$ .

Given that  $C$  with position vector  $\begin{pmatrix} 1 \\ k \end{pmatrix}$  lies on  $AB$  produced, calculate the value of  $k$  and the value of  $|2\overline{AB} + \overline{OC}|$ . (6 marks)

- (b) Find the exact value of  $\sin \frac{\pi}{12} \cos \frac{\pi}{6} \tan \frac{7\pi}{12}$ . (7 marks)

7. (a) A rectangular box has a square base of side  $x$  cm. If the sum of one side of the square and the height is 12 cm, express the volume of the box in terms of  $x$ . Use this expression to determine the maximum volume of the box. (6 marks)

- (b) If  $y = \cos^2 3x$ , prove that  $\frac{d^2y}{dx^2} + 36y = 18$ . By using this result show that, if  $z = \sin^2 3x$ ,

$$\text{then } \frac{d^2z}{dx^2} + 36z = 18.$$

(7 marks)