DEFENCE SERVICES TECHNOLOGICAL ACADEMY

ENTRANCE EXAMINATION

MATHEMATICS **Time Allowed: (2) hours**

1. Choose the correct or the most appropriate answers for each question. Write only the letter of the answer. **(30-Marks)**

(1) $f: R \mapsto R$, $g: R \mapsto R$, $f(x) = 3x - 2, g(x) = x^2$, f maps g (x) onto 7, then x =

A. 7 B. 3 C. ± 7 D. ± 3 E. $\pm\sqrt{3}$

(2) Given that $f(x) = \dfrac{13}{2-5x}$, $x \neq \dfrac{2}{5}$, then $f^{-1}(-1)$

A. 5 B. 4 C. 3 D. 2 E. 1

(3) Let f : R $\rightarrow$ R and g: R$\rightarrow$R be functions and given that f(2) = 1 and g (1) = 3. Then $(f^{-1}.g^{-1})(3)$ =

A. -1 B. 0 C. 1 D. 2 E. 3

(4) The remainder when $(2x-1)^3 + 6(3+4x^2) - 10$ is divided by $2x+1$ is

A. 14 B. 12 C. 10 D. -14 E. -12

(5) $x+1$ is a factor of $(4p-3)x^2 + 2px + p^2$ then the values of p are

A. 1,3 B. $-1,-3$ C. $1,-3$ D. $-1,3$ E. 2,3

(6) The expression $x^3 + x^2 - 11x + 4$ has a factor

A. $x-4$ B. $x-2$ C. $x+2$ D. $x+1$ E. $x+4$

(7) The middle term in the expansion of $\left(1 - \dfrac{3}{x}\right)^4$ is

A. $54/x^2$ B. $-54/x^2$ C. $-108/x^3$ D. $108/x^3$ E. $12/x$

(8) The sum of all the coefficients of the terms in the expansion of $(x-1)^{26}$ is

A. -2 B. 2 C. 1/ 2 D. $-1/2$ E. 4

(9) The solution set in R for the inequation $x^2 - 4x + 4 < 0$ is

A. $\{x / x < 2\}$ B. $\{x / x > 2\}$ C. $\{2\}$ D. R E. ϕ

(10) The solution set in R of $x^2 - 4x - 5 > 0$ is

A. $\{x / -1 < x < 5\}$ B. ϕ C. $\{x / x < -1\}$ D. $\{x / x > 5\}$ E. $\{x / x < -1 \text{ or } x > 5\}$

(11) The 6th term of an AP is 17 and the 10th term is 29. Then the 16th term is

A. 40 B. 42 C. 44 D. 46 E. 47

(12) For two numbers *a* and *b*, the A.M is 3 and G.M is $2\sqrt{2}$. Then $a^2 + b^2 =$

A. 36 B. 16 C. 20 D. 6 E. 24

(13) If the n^{th} term of a G.P, is defined by $u_n = \dfrac{1}{9^n}$, then the sum to infinity of the G.P is

A. 9 B. 0.9 C. 1.25 D. 0.125 E. 0.25

(14) If $\begin{bmatrix} a & 3 \\ 8 & b \end{bmatrix}$ is singular and a, b are positive integers then a + b cannot be.

A. 10 B. 11 C. 14 D. 18 E. 25

(15) If A is a 2 x 2 matrix such that det A = k and p is a real number then det (pA) =

A. pk B. pk^2 C. p^2k D. p^2k^2 E. p

(16) Let A be an event such that p(A will occur) = p(A will not occur) = q and $0 < p < 1$. If the sum to infinity of the G.P $p + p^2 + p^3 + - - -$ is x, then the sum to infinity of the G.P $q + q^2 + q^3 + - - -$ is

A. $1-x$ B. $1+x$ C. 1/ x D. 1/ (1-x) E. 1/ (1+x)

(17) A and B are two students such that P(both A and B will pass the examination) is 1/ 4 and P(A will pass but B will fail the examination) is 2/ 3. Then P(A will pass the examination) is

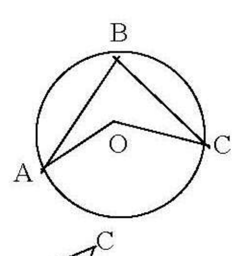
A. 1/ 6 B. 1/ 12 C. 1/ 2 D. 3/ 4 E. 11/ 12

(18) A and B are two events such that $P(A) + P(\text{not } B) = \dfrac{6}{7}$. Then $P(\text{not } A) + P(B) =$

A. 1/ 7 B. 3/ 7 C. 5/ 7 D. 8/ 7 E. 11/ 7

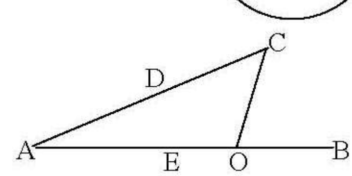
(19) In circle O, AB = BC, $\angle$ ABC = 40°. Then $\angle$ OAB is

A. 20° B. 30° C. 40° D. 50° E. 60°

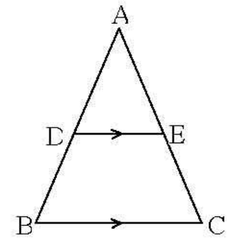


(20) In the figure, $AE = 2$, $AD = 3$, $DC = 5$. Then $OC =$

A. 4 B. 5 C. 6 D. 7 E. 8



(21) In the figure, DE // BC and $\alpha(\text{BCED}) = \frac{1}{2}\alpha(\Delta\,\text{ABC})$. Then AD: DB =

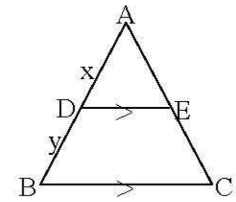


A. 1 : 1 B. 1 : 2C. 1 : $\sqrt{2}$ D. 1 : 4 E. 1: $(\sqrt{2}-1)$

(22) A side of one of the similar triangles is 5 times as long as the corresponding side of the other. If the area of the smaller triangle is 6 sq cm, the area of the larger one, in sq cm is.

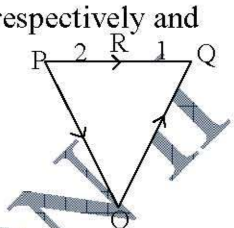
A. 25 B. 150 C. 6 D. 36 E. 121

(23) In the figure, if $\alpha(\Delta ADE):\alpha(BCED) = 4:21$, then $x : y =$



A. 5:2 B. $2:\sqrt{21}$ C. 2:5 D. 4:25 E. 2:3

(24) If the position vectors of P and Q with respect to origin O are $-2\hat{i}+7\hat{j}$ and $4\hat{i}-5\hat{j}$ respectively and $PR:RQ = 2:1$, then $\overrightarrow{RQ} =$



A. $2\hat{i}-4\hat{j}$ B. $-2\hat{i}+4\hat{j}$ C. $3\hat{i}-4\hat{j}$

D. $-3\hat{i}+\hat{j}$ E. $\hat{i}-2\hat{j}$

(25) In ΔABC if $\angle A:\angle B:\angle C = 1:2:3$ then $a:b:c =$

A. $1:2:\sqrt{3}$ B. $1:\sqrt{3}:2$ C. $2:1:\sqrt{3}$ D. $2:\sqrt{3}:1$ E. $\sqrt{3}:2:1$

(26) $\operatorname{cosec}\theta$ cannot be A. 0 B. -100 C. $\sqrt{5}$ D. -2.7 E. 4

(27) If $0^\circ \le \theta \le 360^\circ$, the number of elements in the solution set of $\sin\theta = 0$ is

A. 0 B. 1 C. 2 D. 3 E. 4

(28) The stationary point of the curve $y = 2x - x^2$ is

A. $(-1,1)$ B. $(1,-1)$ C. $(-1,-1)$ D. $(-1,0)$ E. $(1,1)$

(29) The gradient of the curve $y = 3x^2 - 4x + 9$ at the point $(1, 8)$ is

A. -1 B. -2 C. -3 D. 2 E. 3

(30) $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} = ?$ A. 0 B. 1 C. 2 D. -1 E. -2

2. Two functions f and g are defined by $f : x \mapsto 3x+4$, $g : x \mapsto x^2+6$. Using this notation, obtain expressions for $f.g$ and $g.f$. Find the values of x for which (i) $f = g$, (ii) $f.g = g.f$

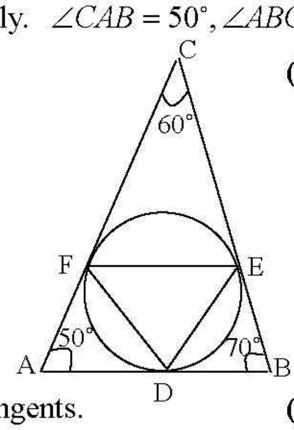
(10-Marks)

3. The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 2 and the roots of the equation $f(x) = 0$ are 1/ 2, 1 and k. Given that $f(x)$ has remainder of 15 when divided by $x-2$, find (i) the value of k, (ii) the remainder when $f(x)$ is divided by $x+1$. **(10-Marks)**

4. The first term of an A.P is 2. The sum of the first 8 terms is 58 and the sum of the whole series is 325. Calculate (i) the common difference, (ii) the number of terms, (iii) the last terms. **(10-Marks)**

5. Each of three tennis players A, B and C play against each other exactly once. In these games the probability that A beats B is 1/ 2 and the probability that B beats C is 1/ 3 and the probability that C beats A is 1/ 4. Find the probability that (i) A wins both games, (ii) B wins no games, (iii) A wins both game and B wins no game (iv) C wins exactly one game. **(10-Marks)**

6. AB, BC and CA are tangents to the circle at D, E and F respectively. $\angle CAB = 50^\circ, \angle ABC = 70^\circ$ and $\angle BCA = 60^\circ$. Find (i) $\angle ADF$ (ii) $\angle BED$ (iii) $\angle DEF$ **(10-Marks)**



7. Prove that cosec θ + cot θ = cot $\frac{\theta}{2}$. Hence

(i) deduce the value of cot 15° in surd form,

(ii) express cosec θ + cosec 2θ + cosec 4θ as the difference of two cotangents. **(10 Marks)**

8. The curve $y = ax + \frac{3}{x}$ has gradient 1 at $x = 2$. Find the value of a and the x-coordinate of another point at which the gradient is 1. **(10-Marks)**